Time: 1 hr 30 min **Marks: 40**

1. Check whether the relation R on the set given by $R=\{(1,1),(2,2),(2,3),(1,2),(3,3)\}$ is an equivalence relation. **(1)**
2. Check whether the relation S on the set R defined by $S=\left\{(a,b): a\le b^3\right\}$ is reflexive, symmetric or transitive. **(1)**
3. Verify whether the modulus function defined $f:R\to R$ is one-one and on-to. **(2)**
4. Verify $f:N\to N$ defined by $f(n)=\begin{cases}\frac{n+1}{2}, n=odd\\ \frac{n}{2}, n=even\end{cases}$ is injective and surjective. **(2)**
5. Give an example of a relation, which is (i) Transitive but neither reflexive nor symmetric (ii) reflexive and symmetric but not transitive. **(2)**
6. **If** $f:R\to R$ be given by $f(x)=x^2+2$ and $g:R-\{1\}\to R$ be given by $g(x)=\frac{x}{x-1}$. Find $f\circ g$ and $g\circ f$. **(2)**
7. Let R be a relation defined on the set of natural numbers N as $R=\left\{(x,y): 2x+y=41\right\}$. Find the domain and range of R. Also verify whether R is reflexive, symmetric or transitive. **(3)**
8. Prove that the relation R on the set NxN defined by $(a,b)R(c,d)\Leftrightarrow a+d=b+c$, $\forall (a,b),(c,d)\in N\times N$ is an equivalence relation. Also find $[(2,3)]\, and\, [(1,3)]$. **(4)**
9. Let N denote the set of all natural numbers and R be the relation on NxN defined by $(a,b)R(c,d)\Leftrightarrow ad(b+c)=bc(a+d)$, check whether R is an equivalence relation on NxN. **(4)**
10. If R and S are two equivalence relations on set A, then prove that $R\cap S$ is an equivalence relation on A. Is union of two equivalence relation on a set a equivalence relation? Justify. **(4)**
11. Prove that the relation R on the set Z defined by $(a,b)\in R\Leftrightarrow a-b$ is divisible by 5 is an equivalence relation. **(3)**
12. Show that the relation R on the set $A=\{x\in Z: 0\le x\le 12\}$ given by $(a,b)\in R\Leftrightarrow |a-b|$ is a multiple of 4. Find the set of all elements related to 1. **(3)**
13. Show that $f:R-\{3\}\to R-\{1\}$ given by $f(x)=\frac{x-2}{x-3}$ is bijective. **(3)**
14. Show that if $f:A\to B$ and $g:B\to C$ are one-one then $g\circ f:A\to C$ also one-one. Are f and g both necessarily one-one, if $g\circ f$ is one-one? Justify. **(3)**
15. Show that if $f:A\to B$ and $g:B\to C$ are on-to then $g\circ f:A\to C$ also on-to. Are f and g both necessarily on-to, if $g\circ f$ is on-to? Justify. **(3)**
